TOPICS COVERED BY THE MATHEMATICS APTITUDE TEST FOR ADMISSION AND PLACEMENT

1. Real number
2. Polynomials
3. Inequalities
4. Absolute Values
5. Functions
6. Applications (1): Areas, volumes, weights
7. Applications (2): Ratio and proportions, percentages
8. Problem Solving Strategies

- **The detailed topics and some examples are provided on the following pages.**

TOPICS COVERED BY THE MATHEMATICS PLACEMENT TEST

1. REAL NUMBERS

A number of the form $\frac{a}{b}$, $a, b \in \Re$, $b \neq 0$ is called a fraction.

$\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.

Properties of Fractions

1) $\frac{a.k}{b.k} = \frac{a}{b}$, $b \neq 0$ 2) $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$ 3) $\left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{ad}{bc}$

4) $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ 5) $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$ 6) $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$

Example

$$\frac{\frac{3}{4}+\frac{2}{5}}{\frac{2}{3}-\frac{3}{5}} = \frac{\frac{15+8}{20}}{\frac{10-9}{15}} = \frac{\frac{23}{20}}{\frac{1}{15}} = \left(\frac{23}{20}\right)\left(\frac{15}{1}\right) = \frac{23.15}{20} = \frac{69}{4}.$$

Exponents:-

We define for $n \in N$, $x^n = x.x \cdots .x$, where the factor x occurs n times. x is called the base and n is the exponent.

$$x^0 = 1 \text{ and } x^{-n} = \frac{1}{x^n}, \quad x \neq 0.$$

Laws of exponents if $p, q, r \in Z$, then

(i) $a^p a^q = a^{p+q}$ (ii) $\frac{a^p}{a^q} = a^{p-q}$ (iii) $(a^p)^r = a^{pr}$

(iv) $(ab)^p = a^p b^p$ (v) $\left(\frac{a}{b}\right)^p = \frac{a^p}{a^q}$

where a and b are real numbers such that the above expressions are defined.

Example

$$\begin{aligned} & \left(2ab^{-3}\right)^2\left(-2a^{-1}b^3\right)^2 \\ = & \left(2ab^{-3}.(-2)a^{-1}b^3\right)^2 \\ = & \left(-4\frac{a}{b^3}.\frac{b^3}{a}\right)^2 = (-4)^2 = 16. \end{aligned}$$

Definition:- $a^{\frac{1}{n}} = \begin{cases} b & \text{if and only if } a = b^n, \ n \text{ is odd} \\ |b| & \text{if and only if } a = b^n, \ n \text{ is even and } a \geq 0. \end{cases}$

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m \text{ or } a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}.$$

Example

$$\begin{aligned} 16^{\frac{3}{2}} &= \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64 \\ 16^{\frac{3}{2}} &= \left(16^3\right)^{\frac{1}{2}} = (4096))^{\frac{1}{2}} = 64. \end{aligned}$$

Definition:- If $n \in Z^+$ and $b \in \Re$ such that $b^{\frac{1}{n}} \in \Re$ then we define $\sqrt[n]{b} = b^{\frac{1}{n}}$.

Properties of radicals $a, b \in R$ and $m, n \in N$

(i) $\left(\sqrt[n]{a}\right)^n = a$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ (iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iv) $\sqrt[n]{a^n} = \begin{cases} |b|, & n \text{ is even} \\ a, & n \text{ is odd} \end{cases}$ (v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Example Simplify

(i) $\sqrt{12} + \sqrt{75} = \sqrt{(4)(3)} + \sqrt{(25)(3)} = \sqrt{4}\sqrt{3} + \sqrt{25}\sqrt{3} = 2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

(ii) $\sqrt{\frac{162}{49}} = \frac{\sqrt{(2)(81)}}{\sqrt{49}} = \frac{9\sqrt{2}}{7}$

Example Rationalize the denominator $\frac{2}{\sqrt{3}+\sqrt{5}}$

$$\frac{2}{\sqrt{3}+\sqrt{5}} = \frac{2}{\sqrt{3}+\sqrt{5}} \cdot \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}} = 2\frac{\left(\sqrt{3}-\sqrt{5}\right)}{\left(\sqrt{3}\right)^2 - \left(\sqrt{5}\right)^2} = 2\frac{\sqrt{3}-\sqrt{5}}{3-5} = \sqrt{5}-\sqrt{3}.$$

2. POLYNOMIALS

Definition:- a polynomial is an expression in the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where the $a's$ are real numbers and n is a non negative integer. The $a's$ are called the coefficient of the polynomial, and n the degree of the polynomial.

Examples

(1) $\sqrt{x}$ is not a polynomial since $\sqrt{x} = x^{\frac{1}{2}}$ and $\frac{1}{2}$ is not a positive integer

(2) $\frac{3}{2}x - 5$ is a first degree polynomial in x.

Factor Formulas

$x^2 - a^2 = (x-a)(x+a)$	difference of two squares
$x^3 - a^3 = (x-a)\left(x^2 + ax + a^2\right)$	Difference of two cubes
$x^3 + a^3 = (x+a)\left(x^2 - ax + a^2\right)$	Sum of two cubes
$(x \pm a)^2 = x^2 \pm 2ax + a^2$	
$(x \pm a)^3 = x^3 \pm 3ax^2 + 3a^2x \pm a^3$	

Rational Expressions

Definition:- Arational expression is an expression of the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials.

Examples $\frac{x}{x+1}, \frac{x^2-3}{x^3+x+1}, \frac{x^6}{x^2+1}$ are rational expressions

Reduced form: a rational expression is in reduced form if its numerator and denominator have no (nontrivial) common factor.

For example, $\frac{x}{x+2}$ is in reduced form, but $\frac{x^2}{x^2+2x}$ is not, since

$$\frac{x^2}{x^2+2x} = \frac{x.x}{x(x+2)} = \frac{x}{x+2}.$$

To reduce a rational expression, we factor numerator and denominator and divide out, or cancel common factor.

Examples

$$(1) \quad \frac{x^2+7x+10}{x^2-25} = \frac{(x+5)(x+2)}{(x+5)(x-5)} = \frac{x+2}{x-5}.$$

$$(2) \quad \frac{x}{3x-6} - \frac{2}{2-x} = \frac{x}{3(x-2)} - \frac{2}{2-x}$$

$$= \frac{x}{3(x-2)} - \frac{2}{-(x-2)} = \frac{x}{3(x-2)} + \frac{2}{x-2}$$

$$= \frac{x}{3(x-2)} + \frac{6}{3(x-2)} = \frac{x+6}{3(x-2)}$$

$$(3) \quad \frac{x+2}{\frac{x^2-4}{x}} = \frac{x+2}{1} \cdot \frac{x}{x^2-4} = \frac{(x+2)x}{(x-2)(x+2)} = \frac{x}{x-2}.$$

Solving equations

1) linear equation: $ax+b=0, \quad a \neq 0.$

Example $(i) \quad 5x+3=0 \Rightarrow x = -\frac{3}{5}$

$$(ii) \quad \frac{x}{3} + \frac{3x}{4} = 2 \Rightarrow 4x+9x=24$$

$$\Rightarrow 13x = 24 \Rightarrow x = \frac{24}{13}$$

$$(iii) \quad \frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4} \quad \text{mult. by LCD } (x^2-4)$$

$$x+2 = 3(x-2) - 6x \qquad x \neq \pm 2$$

$$x+2 = -6-3x$$

$$4x = -8$$

$$x = -2$$

(Since x cannot $= -2$ in the original equation, $\Rightarrow$ the equation has no solution. ($x=-2$ is called an extraneous solution).

2) Quadratic Equation

An equation of the form $ax^2+bx+c=0, \quad a \neq 0$ is called a quadratic equation.

Methods for solving

(i) Factoring (ii) Completing the square (iii) Quadratic formula

Note: if $a.b = 0 \Rightarrow$ either $a = 0$ or $b = 0$.

example of completing the square

(i) Solve $x^2 + 2x = 4$

$x^2 + 2x + 1 = 4 + 1$ (take half the coefficient of

$(x+1)^2 = 5$ x square it and add it to both sides)

$x + 1 = \pm\sqrt{5}$

$x = -1 \pm \sqrt{5}$

Quadratic Formula

$$\text{if } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. INEQUALITIES:-

Properties : let $a, b, c \in R$

(i) if $a < b$ and $b < c$ then $a < c$. (ii) if $a < b$ then $a + c < b + c$.

(iii) if $a < b$, then $a - c < b - c$. (iv) if $a < b$, and $c > 0$, then $ac < bc$.

(v) if $a < b$, and $c < 0$, then $ac > bc$. (vi) if $a < b$, and $ab > 0$, then $\frac{1}{a} > \frac{1}{b}$.

(vii) if $0 < a < b$ and $n \in N$, then $a^n < b^n$.

(viii) if $0 < a < b$ and $n \in N$, then $\sqrt[n]{a} < \sqrt[n]{b}$.

(ix) if $a < b$ and $c < d$, then $a + c < b + d$.

The other relations $\leq, >, \geq$ satisfy similar properties to the above

Linear Inequality:

Example Solve

$3x - 11 < x + 5$

$3x - x < 5 + 11$

$2x < 16$

$x < 8$

Solution set is $S = (-\infty, 8)$.

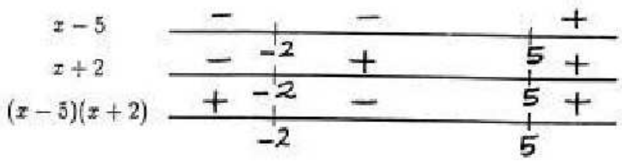
Quadratic Inequality

Example Solve

$x^2 > 3x + 10$

$x^2 - 3x - 10 > 0$

$(x-5)(x+2) > 0$



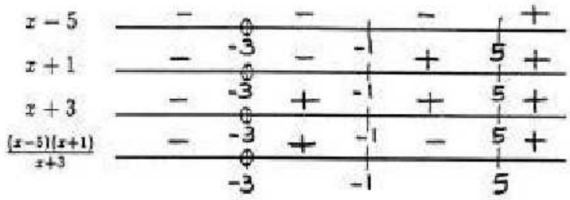
Solution Set $S = (-\infty, -2) \cup (5, \infty)$

Rational Inequality

Example Solve

$$\frac{x^2 - 4x - 5}{x + 3} > 0$$

$$\frac{x^2 - 4x - 5}{x + 3} > 0 \Rightarrow \frac{(x-5)(x+1)}{x+3} > 0$$



The solution set is $S = (-3, -1) \cup (5, \infty)$

4. ABSOLUTE VALUE

Definition:-

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$\sqrt{a^2} = |a|$

Properties of absolute value: let a and b be real numbers.

(*i*) $|a| \geq 0$ (*ii*) $|-a| = |a|$

(*iii*) $|ab| = |a||b|$ (*iv*) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $b \neq 0$

(*v*) $|a| = b$ if and only if $a = b$ or $a = -b$

(*vi*) $|a| < b$ if and only if $-b < a < b$ $(b > 0)$

(*vii*) $|a| > b$ if and only if $a > b$ or $a < -b$

(*viii*) $-|a| \leq a \leq |a|$

Equations Involving Absolute Value

Examples Solve

(1) $|x - 3| = 4$. from property (v)

$x - 3 = 4$ or $x - 3 = -4$

$\Rightarrow x = 7$ or $x = -1$

(2) $|3x-2| = |5x = 4|$

$3x-2 = 5x+4$ or $3x-2 = -(5x+4)$

$\Rightarrow x = -3$ and $x = -\frac{1}{4}$.

Inequalities Involving Absolute Value

Example Solve

$$\frac{1}{|2x-3|} > 5, \quad x \neq \frac{3}{2}$$

$$\begin{aligned} |2x-3| &< \frac{1}{5} \quad \left(a < b \text{ and } ab > 0 \Rightarrow \frac{1}{a} > \frac{1}{b}\right) \\ -\frac{1}{5} &< 2x-3 < \frac{1}{5} \\ 3-\frac{1}{5} &< 2x < \frac{1}{5}+3 \\ \frac{14}{5} &< 2x < \frac{16}{5} \\ \frac{7}{5} &< x < \frac{8}{5} \qquad x \neq \frac{3}{2}. \end{aligned}$$

Solution Set is

$$\left(\frac{7}{5}, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \frac{8}{5}\right)$$

5. FUNCTIONS

Let A and B be two non-empty sets. A function $f : A \Rightarrow B$ is a rule that assigns to each element in A exactly one element in B.

The set A is called the domain of f.

Operations on Functions

Let f and g be two functions with domains D_f and D_g respectively. Then

$$D_{f+g} = D_{f-g} = D_{fg} = D_f \cap D_g$$

and

$$D_{\frac{f}{g}} = \{x : x \in D_f \cap D_g, g(x) \neq 0\}$$

Composition of Functions

Given two functions f and g, the composition of f and g (written fog) is defined $(fog)(x) = f(g(x))$.

Example Given

$$f(x) = \frac{x}{x+1} \text{ and } g(x) = \frac{2}{x-1}, \text{ find } fog \text{ and its domain}$$

$$(fog)(x) = f(g(x)) = f\left(\frac{2}{x-1}\right) = \frac{\frac{2}{x-1}}{\frac{2}{x-1}+1}$$

$$= \frac{\left(\frac{2}{x-1}\right)(x-1)}{\left(\frac{2}{x-1}+1\right)(x-1)} = \frac{2}{2+(x-1)} = \frac{2}{x+1}.$$

$D_g = \{x : x \neq 1\}$. Now $-1 \in D_g$ and $g(-1) = -1$. Since $-1 \notin D_f$ it follows that -1 can not be included in the domain of fog. Thus

$$D_{fog} = \{x : x \neq \pm 1\}$$

6. APPLICATIONS (1)

Areas, Volumes, Weights

1. Areas

Rules : a) Area of a triangle $= \frac{1}{2}$ (base) (height)

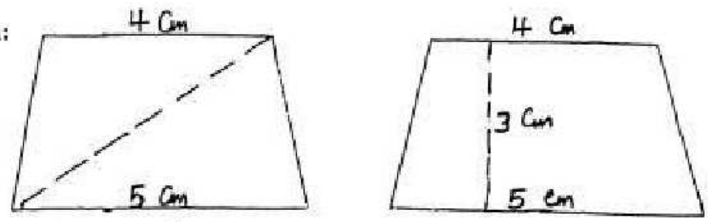
b) Area of a rectangle = (length) (width)

c) Area of a circle $= \pi r^2$, r = radius.

Using the above rules one can find the areas of "little more complicated" shapes as shown in the following examples:

Example 1. Find the area of the trapezoid shown in the figure

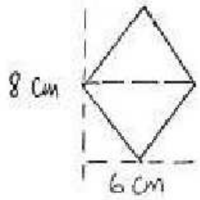
Solution:



the trapezoid can be divided into two triangles as shown. Those triangles have bases 4cm and 5cm respectively, and have the same height 3cm. So Area $= \frac{1}{2}(4)(3) + \frac{1}{2}(5)(3) = 13.5\ cm^2$.

Remark: One can use the formula for the area of a trapezoid too.

Example 2. Find the area of the rhombus shown in the figure.



Solution: We divide the rhombus into two congruent triangles as shown.

$$\text{Area} = 2\left(\tfrac{1}{2} \times 6 \times 4\right) = 24 \text{ cm}^2$$

Remark: One can use the formula for the area of a rhombus too.

Example 3: Find the area of the region shown in the figure.

Solution: We divide the region into a rectangle and a square as shown.

$$\text{Area} = (4 \times 6) + (2 \times 2) = 24 + 4 = 28 cm^2$$

Example 4. Find the area of the region shown in the figure.

Solution:

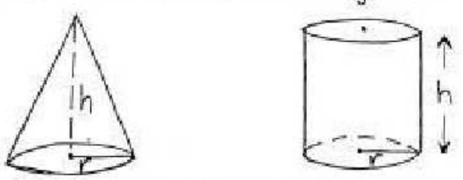
$$\begin{aligned} \text{Area} &= \text{area of rectangle} + \text{Area of semicircle} \\ &= 6 \times 4 + \frac{1}{2}\pi(2^2) \\ &= 24 + 2\pi \ \ cm^2. \end{aligned}$$

2. Volume

Rules : a) Volume of a right prism = Area of base × Aptitude

b) Volume of a right cylinder $= \pi r^2 h$

c) Volume of a right cone $= \frac{1}{3}\pi r^2 h$,.



Students will not be asked to find volumes, but this will be a part of a word problem.

Example 1. A road 3km long and 14m wide to be covered with tar 30cm thick. How many trucks of tar are needed if the capacity of each truck is 12 m^3?

Solution: Amount of tar needed $= 3000 \times 14 \times 0.30 = 12600\ m^3$
number of trucks $= 12600 \div 12 = 1050$.

Example 2. 128 Liters of orange juice to be distributed into bottles the capacity of which is $\frac{1}{4}$ Liter. How many bottles are needed ?

Solution: $128 \div \frac{1}{4} = 128 \times 4 = 512$ bottles

Example 3. In a dairy, milk is sterilized in cylindrical barrels. The diameter of the base of each barrel is one meter and the altitude is 2m. Milk is then distributed into cans the capacity of which is 314 cm^3 per can. How many cans of milk does each barrel give? (use $\pi = 3.14$).

Solution:

$$\text{Volume of each barrel} = \pi r^2 h = 3.14 \times (50)^2 \times 200 = 1570000\ cm^3$$

$$\text{number of cans needed} = 1570000 \div 314 = 5000$$

3. Weights

These would appear within word problems

Example 1. The weight of Salim and Shahid together is 120 kg. What is the weight of each if Salim is 24kg heavier than Shahid?

Solution:

$$120 - 24 = 96$$

$$\text{weight of Shahid} = 96 \div 2 = 48\ kg.$$

$$\text{weight of Salim} = 48 + 24 = 72\ kg.$$

Example 2. The weight of a cat, a dog and a lamb together is 28 kg. What is the weight of each if the weight of the cat is half that of the dog and the weight of the dog is half that of the lamb?

Solution: The dog is twice the weight of the cat, and the lamb is four times the weight of the cat. So weight of the cat is $28 \div 7 = 4$ kg.
weight of the dog $= 4 \times 2 = 8$ kg.
weight of the lamb $= 4 \times 4 = 16$ kg.

4. Changing Units:

a) the change should be within the metric system. If a change involves more than one system (from imperial to metric for example), then the relevant information must be given.

b) The questions can be direct as in the examples below, or within word problems as in examples (1) and (3) in the volumes section above.

Example 1. If one pound = 454 grams then:

a) 350 kg = $\cdots\cdots$ pounds

b) 105 g = $\cdots\cdots$ pounds

c) 13 pounds = $\cdots\cdots$ kg.

Solution:

a) $350 \div 0.454 = 770.9$ pounds

b) $105 \div 454 = 0.23$ pounds

c) $13 \times 0.454 = 5.9\ kg$.

Example 2. The speed of a car is 60 km/hour. Find its speed in m/minute.

Solution: $\frac{60 \times 1000}{60} = 1000\ m/minute$

Example 3. Fill in the space in what follows:

a) $3\ m^3 = \cdots\cdots cm^3$

b) 200 liters $= \cdots\cdots m^3$, given that 1 liter $= 1000\ cm^3$

c) $7\ m^2 = \cdots\cdots cm^2$

d) $150000\ cm^2 = \cdots\cdots m^2$

Solution:

a) $3 \times 10^6 = 3000000\ cm^3$

b) $200 \times 1000 = 200000\ cm^3 = 0.2\ m^3$

c) $7m^2 = 7 \times 10^4 = 70000\ cm^2$

d) $150000\ cm^2 = 150000 \div 10^4 = 15\ m^2$.

APPLICATIONS (2)

Ratio and Proportion, Percentages

1. Ratio and Proportion:

a) Ratios compare the sizes of related quantities. For example if the ratio of my income to my brother's income is 3:2 then this means that for every 2 K.D. that my brother earns, I earn 3 K.D. So if my brother gets 200 K.D., I obtain 300 K.D. and so on.

Rule: A ratio can be multiplied or divided throughout by the same number. For example, the ratio 3:2 is the same as the ratio 300:200. Thus we can treat the ratio 3:2 as the fraction $\frac{3}{2}$.

Example 1. If the ratio of games won to those lost is 15:16 and the number of games lost is 64, find the number of games that were won.

Solution:

$$\frac{w}{\ell} = \frac{15}{16}, \text{ so } \frac{w}{64} = \frac{15}{16}, \quad 16w = 15 \times 64$$
$$w = \frac{15 \times 64}{16} = 60.$$

So the number of games won is 60.

Example 2. Given the same situation as in example 1. Find the number of games played.

Solution: From example 1, 64 games were lost and 60 games were won. So the number of games is $60 + 64 = 124$.

Example 3. The total daily wage of three workers is 72 K.D. distributed in the ratio 3:4:5. What is the daily wage of each of them?

Solution: The number of shares is $3 + 4 + 5 = 12$

Each share is worth $72 \div 12 = 6$ K.D.

The first worker takes $6 \times 3 = 18$ K.D.

The 2nd worker takes $6 \times 4 = 24$ K.D.

The 3rd worker takes $6 \times 5 = 30$ K.D.

b) If two varying quantities are always in the same ratio, we say that they are directly proportional to one another (or, simply, proportional).

For example, if we buy a number of identical pens, then the price of the pens is proportional to its number, because $\frac{\text{price of pens}}{\text{number of pens}}$ = constant = price of one pen.

Example 1. If the price of 11 pens is 33 K.D., what is the price of 15 pens?

Solution: Price of 15 pens $= \frac{33 \times 15}{11} = 45$ K.D.

Example 2. If the weight of 16 cm^3 of a metal is 24g, what is the weight of 20 cm^3 of that metal?

Solution: Weight of $20cm^3 = \frac{24\times20}{16} = 30$ g.

c) Two quantities are said to be inversely proportional if one of them is proportional to the inverse or reciprocal of the other. For example, the number of workers needed to do a job is inversely proportional to the time needed to finish the job. That is if the number increases, the time needed decreases.

Example 1. Four workers need ten days to paint a certain wall. How many days are needed for 5 workers to paint the same wall?

Solution: One worker needs $4 \times 10 = 40$ days
Five workers need $40 \div 5 = 8$ days

Example 2. Eleven taps fill a tank in three hours. How long would it take to fill the tank if only six taps are working?

Solution: One tap needs $11 \times 3 = 33$ hours
6 taps need $33 \div 6 = 5$ hours and 30 minutes.

2. Percentages

The symbol 13% means 13 out of 100. So as a fraction it is $\frac{13}{100}$, and as a decimal it is 0.13. So if we want 13% of, say, 500 K.D. we multiply $500 \times \frac{13}{100} = 65K.D.$

Example 1. Out of a class of 25 students, 72% are good in Arabic Find the number of students that are not good in Arabic.

Solution: number of students that are good $= 25 \times \frac{72}{100} = 18$ so those that are not good in Arabic are $25 - 18 = 7$ student.

Example 2. Ibrahim's weight now is 8% more than it was last year. If his weight last year was 55 kg, what is his weight now?

Solution: The increase in weight is $55 \times \frac{8}{100} = 4.4$ kg.

This weight now is $55 + 4.4 = 59.4$ kg.

Example 3. When water freezes, its volume increases by 4%. What volume of water is required to make 728 cm^3 of ice?

Solution: Volume of ice is 104% of the volume of water. So volume of water needed is $\frac{728 \times 100}{104} = 700\ cm^3$.

3. Interest

Many institutions such as banks offer savings accounts. If you put money into such an account, the bank uses your money and pays you for that use. The amount that the bank pays you is called the interest. The amount of money you put in the bank is called the capital. Interest on money borrowed (or lent) is usually given as a percentage of the capital per year. This is called the interest rate.
Rule: Interest = Capital × interest rate × time(in year).

Example 1. A man put 5000 K.D. in a savings account for 3 years with an interest rate of 12%. Waht would be the balance of his account by the end of this period?

Solution: Interest $= 5000 \times \frac{12}{100} \times 3 = 1800$ K.D.
Balance of his account will be $5000 + 1800 = 6800 K.D.$

Example 2. A man put 2800 K.D. for a year in a savings account. If the interest was 238 K.D. what was the interest rate?

Solution: interest rate is $\frac{238 \times 100}{2800} = 8.5$
Example 3. Find the amount borrowed if 28750 K.D. has to be repaid after one year when the interest rate is 15%.

Solution: The amount repaid is 115% of the amount borrowed so the amount borrowed is $\frac{28750 \times 100}{115} = 25000 K.D.$

8 Problem Solving Strategies.

Here various exercises are given. No advanced or special mathematics is needed. Only simple arithmetic will be used. Each problem might need several steps to be solved.

Example 1. Find the diameter of a circle as a function of its area.

Solution: $A = \pi r^2$, $r^2 = \frac{A}{\pi}$, $r = \sqrt{\frac{A}{\pi}}$.
diameter $= 2r = 2\sqrt{\frac{A}{\pi}}$.

Example 2. Peter is x years old. How old is his brother that was born y years earlier.

Solution: His brother is $x + y$ years old.

Example 3. A rectangle is x meters long. If its length is 7 meters longer than its width, find the parameter and area of this rectangle.

Solution: width is $x - 7$
Parameter is $2[x + (x - 7)] = 4x - 14$
Area is $x(x - 7)$.

Example 4. A tap fills a tank in two hours. Another tap at the bottom of the tank needs three hours to empty it. How long it takes to fill the tank if both taps were open.

Solution: In one hour the first tap fills $\frac{1}{2}$ the tank.
In one hour the second tap empties $\frac{1}{3}$ the tank.
So if both taps were open, in one hour we have $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ of the tank is filled. Hence 6 hours are needed to fill the tank when both taps are open.

Example 5. A man bought 360 kg. of potatoes for 120 fils per kilo. He found out that 20% of the potatoes is unsuitable for selling. What would be the selling price of one kilo of the rest of the potatoes if he wants a 15% profit?

Solution: The man paid $= 360 \times 120 = 43200$ fils
The profit requited $= 43200 \times \frac{15}{100} = 6480$
The total selling price $= 43200 + 6480 = 49680$ fils
The unsuitable amount of potatoes $= 360 \times \frac{20}{100} = 72$ kg.
The suitable amount of potatoes $= 360 - 72 = 288$ kg.
Selling price of one kilo is $49680 \div 288 = 172.5$ fils.

Example 6. The perimeter of a square is twice the perimeter of an equilateral triangle. If one side of the square is 75 meters, what is the length of one side of the triangle?

Solution: Perimeter of the square = 4 x 75 = 300 m

Perimeter of the triangle = ½ x 300 = 150 m

Side of the triangle = 150 ÷ 3 = 50 m